

A-Level Mathematics Paper 5 (S1)

Notes

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LACAS

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Title A-L MATHEMATICS PAPER 5 (S1) NOTES

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Published by MS Books (042-35774780)

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AKBAR LAW CHAMBERS
39-40, 1st Floor, Sadiq Plaza, The Mall, Lahore.
0307-4299886, 042-36314839

For Complaints/Order **MS Books**
83-B Ghalib Market, Gulberg III Lahore
(042-35774780),(03334504507),(03334548651)

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Representation of data

Syllabus

Candidates should be able to:

- select a suitable way of presenting raw statistical data, and discuss advantages and/or disadvantages that particular representations may have
- draw and interpret stem-and-leaf diagrams, box-and-whisker plots, histograms and cumulative frequency graphs
- understand and use different measures of central tendency (mean, median, mode) and variation (range, interquartile range, standard deviation)
- use a cumulative frequency graph
- calculate and use the mean and standard deviation of a set of data (including grouped data) either from the data itself or from given totals $\sum x$ and $\sum x^2$, or coded totals $\sum(x - a)$ and $\sum(x - a)^2$, and use such totals in solving problems which may involve up to two data sets.

Notes and examples

Including back-to-back stem-and-leaf diagrams.

e.g. in comparing and contrasting sets of data.

e.g. to estimate medians, quartiles, percentiles, the proportion of a distribution above (or below) a given value, or between two values.

MS
BOOKS

Data

Data can be defined as groups of information that represent the qualitative or quantitative attributes of a variable or set of variables, which is the same as saying that data can be any set of information that describes a given entity. Data in statistics can be classified into grouped data and ungrouped data.

Any data that you first gather is ungrouped data. Ungrouped data is data in the raw. An example of ungrouped data is a any list of numbers that you can think of.

DISCRETE DATA

Discrete data can take only exact values, for example

the number of cars passing a checkpoint in 30 minutes,
the shoe sizes of children in a class,
the number of tomatoes on each plant in a greenhouse.

The data are known as raw because they have not been ordered in any way.

Frequency distribution for discrete data

To illustrate the data more concisely, count the number of times each value occurs and summarise these in a table, known as a **frequency distribution**.

Number of snails	0	1	2	3	4	5	
Frequency	3	5	11	8	2	1	Total 30

The frequency distribution can be represented diagrammatically by a vertical line graph or a bar chart. The height of the line or bar represents the frequency.

Vertical line graph to show
number of snails

Bar chart to show
number of snails

CONTINUOUS DATA

Continuous cannot take exact values but can be given only within a specified range or measured to a specified degree of accuracy

Other examples of continuous data are

the speed of a vehicle as it passes a checkpoint,
the mass of a cooking apple,
the time taken by a volunteer to perform a task.

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Frequency distribution for continuous data

To form a frequency distribution of the heights of the 20 children, group the information into classes or intervals. Here are three different ways of writing the same set of intervals.

Height (cm)
$119.5 < h < 124.5$
$124.5 < h < 129.5$
$129.5 < h < 134.5$
$134.5 < h < 139.5$
$139.5 < h < 144.5$

Height (cm)
119.5–124.5
124.5–129.5
129.5–134.5
134.5–139.5
139.5–144.5

Height (to the nearest cm)
120–124
125–129
130–134
135–139
140–144

The values 119.5, 124.5, 129.5, ... are called the **class boundaries** or the **interval boundaries**. The upper class boundary (u.c.b.) of one interval is the lower class boundary (l.c.b.) of the next interval.

Width of an interval

The width of an interval is the difference between the boundaries.

Width of an interval = upper class boundary – lower class boundary /

Often intervals with equal widths are chosen, as in the above illustrations in which each width is 5 cm.

To group the heights it helps to use a tally column, entering the numbers in the first row 133, 136, 120, ... etc. and then the second row. It is a good idea to cross off each number in the list as you enter it. The frequency distribution for the above data should read:

Height (cm)	Tally	Frequency
$119.5 < h < 124.5$		1
$124.5 < h < 129.5$		5
$129.5 < h < 134.5$		7
$134.5 < h < 139.5$		4
$139.5 < h < 144.5$		3
		Total 20

It is important to note that when the data are presented only in the form of a grouped frequency distribution, the original information has been lost. For example you would know that there was one item in the first interval, but you would not know what it was. You would know only that it was between 119.5 cm and 124.5 cm.

BOOK

STEM AND LEAF DIAGRAMS (STEMPLOTS)

A very useful way of grouping data into classes while still retaining the original data is to draw a **stem and leaf diagram**, also known as a **stemplot**.

These are the marks of 20 students in an assignment:

84	17	38	45	47	53	76	54	75	22
66	65	55	54	51	44	39	19	54	72

Notice that the lowest mark is 17 and the highest mark is 84.

In stem and leaf diagrams, all the intervals must be of equal width, so it seems sensible to choose intervals 10–19, 20–29, 30–39, ..., 80–89 for this data.

Take the **stem** to represent the tens and the **leaf** to represent the units.

The first five entries
84, 17, 38, 45 and 47
are represented like this:

Stem (tens)	Leaf (units)
1	7
2	
3	8
4	5 7
5	
6	
7	
8	4

When all the numbers
have been entered the
diagram looks like this:

Stem	Leaf
1	7 9
2	2
3	8 3 9
4	5 7
5	3 4 5 4 1 4
6	6 5
7	6 5 2
8	4

The entries in each leaf are now arranged in numerical order and a key is given to explain the stem and leaf. The final diagram looks like this:

Stem and leaf diagram to show assignment marks

Stem	Leaf
1	7 9
2	2
3	3 8 9
4	5 7
5	1 3 4 4 4 5 //
6	5 6
7	2 5 6
8	4

Key 1 | 7 means 17 marks

The stemplot gives a good idea at a glance of the shape of the distribution. It is easy to pick out the smallest and largest values and to see that the **mode is 54**. It is also obvious that the **modal class is 50–59**.

NOTE: The key is essential in explaining how the stemplot has been formed.

In a stem and leaf diagram, or stemplot

- (a) equal intervals must be chosen,
- (b) a key is essential.