A-Level Mathematics Paper 4

Notes

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Contents

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Mathematical Models in Mechanics

Assumptions and approximations often used to simplify the mathematics involved:

a) rigid body is a particle,

b) no air resistance,

c) no wind,

d) force due to gravity remains constant,

e) *light* pulleys and *light* strings etc. have no mass,

f) rods are uniform - constant mass per unit length,

g) a lamina is a flat object of negligible thickness and of constant mass per unit area,

h) the earth's surface, although spherical, is usually modelled by a plane

i) surface is smooth - no friction.

Assumptions made

- motion takes place in a straight line -
- acceleration is constant
- air resistance can be ignored
- objects are modelled as masses concentrated at a single point (no rotation)

Forces and Equilibrium

Syllabus

- identify the forces acting in a given situation
- understand the vector nature of force, and find and use components and resultants
- use the principle that, when a particle is in equilibrium, the vector sum of the forces acting is zero, or equivalently, that the sum of the components in any direction is zero
- understand that a contact force between two surfaces can be represented by two components, the normal component and the frictional component
- use the model of a 'smooth' contact, and understand the limitations of this model
- understand the concepts of limiting friction and limiting equilibrium, recall the definition of coefficient of friction, and use the relationship $F = \mu R$ or $F \leqslant \mu R$, as appropriate
- use Newton's third law.

Force : **Force is an agent which change or try to change the position of an object**

TYPES OF FORCE

Forces of Attraction

frictional force

Contact Forces

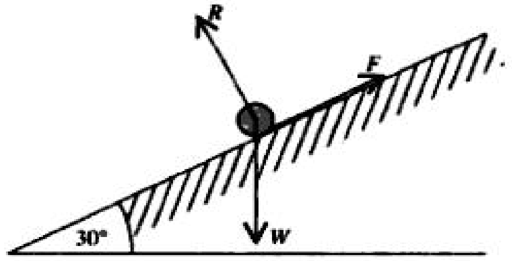
DRAWING DIAGRAMS

When considering any situation concerning the action of forces on a body the first, and vital, step is to draw a clear, uncomplicated diagram of the forces acting on the object.
Some useful points to remember are:

- Unless the object is light, its weight acts vertically downwards.
- If the body is in contact with another surface, a normal reaction always acts on the body. In addition, unless the contact is smooth, there may be a frictional force.
- If the body is attached to another by a string or hinge, a force acts on the body at the point of attachment.

FINDING THE RESULTANT OF COPLANAR FORCES

Consider, for example, a particle resting on a rough plane inclined to the horizontal at 30°. The forces acting on the particle are shown in the diagram.



As the normal reaction and the frictional force are perpendicular to each other, it is sensible to resolve each force in these two directions, i.e. along (↙) and perpendicular (↖) to the plane, i.e.

	Friction	Reaction	Weight
Component ↙	$-F$	0	$W \sin 30°$
Component ↖	0	R	$-W \cos 30°$

Now we can collect the components of force down and perpendicular to the plane and to indicate these operations we write:

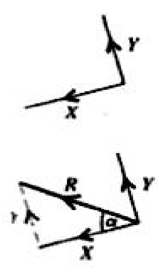
Resolving ↙ gives $W \sin 30° - F$ [1]

Resolving ↖ gives $R - W \cos 30°$ [2]

Calculating the Resultant

If the expression [1] on p. 78 is represented by X and expression [2] by Y, we have

$$X = W \sin 30° - F$$
$$Y = R - W \cos 30°$$



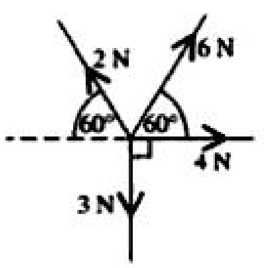
The magnitude of the resultant, R, of X and Y is

$$\sqrt{(X^2 + Y^2)}$$

and R makes an angle α with the plane where

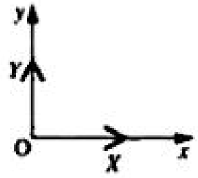
$$\tan \alpha = Y/X$$

Find the resultant of the forces of 4, 6, 2 and 3 newtons shown in the diagram.



Let the resultant have components X and Y newtons in the directions shown.

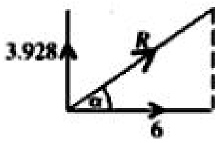
Resolving the forces along Ox and Oy we have:



Resolving $\rightarrow$ $\quad X = 4 + 6\cos 60^\circ - 2\cos 60^\circ$
$= 4 + 3 - 1 = 6$

Resolving $\uparrow$ $\quad Y = 6\sin 60^\circ - 3 + 2\sin 60^\circ$
$= 8 \times 0.8660 - 3 = 3.928$

If the resultant force is R newtons



$$R = \sqrt{(X^2 + Y^2)} = \sqrt{(6^2 + 3.928^2)}$$
$$= 7.17 \quad \text{(3 sf)}$$

and $\quad \tan\alpha = \dfrac{Y}{X} = \dfrac{3.928}{6} = 0.6546\ldots$

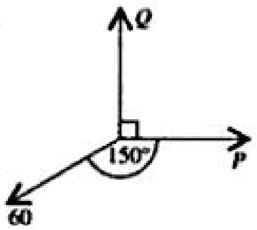
$\Rightarrow \quad \alpha = 33^\circ$ (nearest degree)

Therefore the resultant force is 7.17 N at 33° to the force of 4 N.

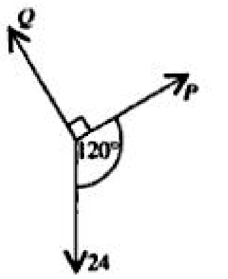
Exercise

In this exercise all forces are measured in newtons.
1 In questions 1 to 3 the forces shown in the diagram are in equilibrium.
Find the values of P, Q and, where appropriate, the value of θ.

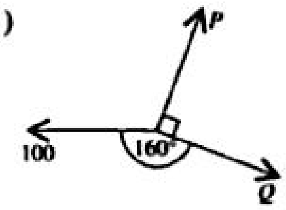
1. (a)



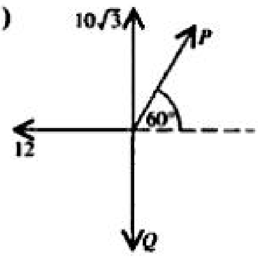
(b)



(c)



2. (a)



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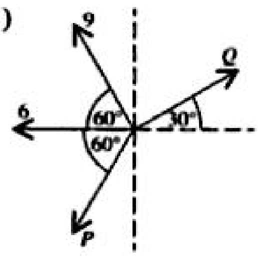

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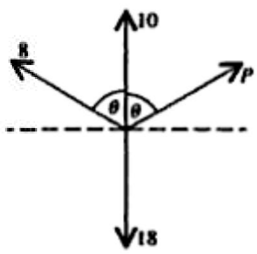
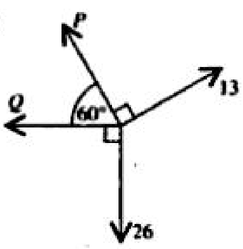


3. (a)



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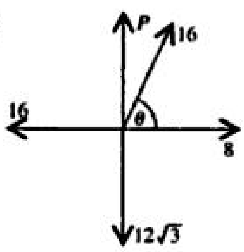

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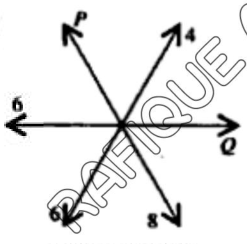

Each angle is 60°

1. (a) $P = 30\sqrt{3}$, $Q = 30$
(b) $P = 12$, $Q = 12\sqrt{3}$
(c) $P = 100\sin 70^\circ = 34.2$, $Q = 100\cos 20^\circ = 94.0$

2. (a) $P = 24$, $Q = 22\sqrt{3}$
(b) $P = 24$, $Q = 15\sqrt{3}$
(c) $P = 8$, $\theta = 60^\circ$

3. (a) $P = 13\sqrt{3}$, $Q = 0$
(b) $\theta = 60^\circ$, $P = 4\sqrt{3}$
(c) $P = 10$, $Q = 8$

The Coefficient of Friction

For two particular surfaces in rough contact, it can be shown experimentally that the limiting value of the frictional force is a fixed fraction of the normal reaction between the surfaces.

This fraction is called the *coefficient of friction* and it is denoted by the Greek letter μ (pronounced mew, in English), i.e. for limiting friction

$$F = \mu R$$

As this is the maximum value of the frictional force, F can take any value from zero up to μR, i.e.

$$0 \leqslant F \leqslant \mu R$$

Once an object begins to move, the frictional force opposing motion remains at the constant value μR. The *marginal* difference between the value of μ when friction is limiting (the coefficient of static friction) and its value once motion takes place (the coefficient of dynamic friction) is so small that at this level it can be ignored.

The value of μ depends upon the materials of which the *two* surfaces in contact are made – it is *not* a property of *one* surface – so ideally we should always refer to *rough contact* rather than to a rough plane, etc.

THE LAWS OF FRICTION

- When the surfaces of two objects are in rough contact, and have a tendency to move relative to each other, equal and opposite frictional forces act, one on each of the objects, so as to oppose the potential movement.
- Until it reaches its limiting value, the magnitude of the frictional force F is just sufficient to prevent motion.
- When the limiting value is reached, $F = \mu R$, where R is the normal reaction between the surfaces and μ is the coefficient of friction for those two surfaces.
- For all rough contacts $0 < F \leqslant \mu R$.
- If a contact is smooth $\mu = 0$.