A-Level Mathematics Paper 3

Notes

<u>Rafique Akthar Baloch</u>

(0300-4897003)

<u>Visiting Teacher AT</u>

LACAS

Title	A-L MATHEMATICS PAPER 3 NOTES
Author	RAFIQUE AKTHAR BALOCH (0300-4897003)
Published by	MS Books (042-35774780)
Legal Advisor	Ashir Najeeb Khan (Advocate) **AKBAR LAW CHAMBERS** 39-40, 1st Floor, Sadiq Plaza, The Mall, Lahore. 0307-4299886, 042-36314839
For Complaints/Order	**MS Books** 83-B Ghalib Market, Gulberg III Lahore (042-35774780),(03334504507),(03334548651)

Contents

PARTIAL FRACTIONS

Syllabus.

Candidates should be able to:	Notes and examples
• recall an appropriate form for expressing rational functions in partial fractions, and carry out the decomposition, in cases where the denominator is no more complicated than – $(ax+b)(cx+d)(ex+f)$ – $(ax+b)(cx+d)^2$ – $(ax+b)(cx^2+d)$	Excluding cases where the degree of the numerator exceeds that of the denominator
• use the expansion of $(1+x)^n$, where n is a rational number and $\lvert x\rvert < 1$.	Finding the general term in an expansion is not included. Adapting the standard series to expand e.g. $\left(2-\frac{1}{2}x\right)^{-1}$ is included, and determining the set of values of x for which the expansion is valid in such cases is also included.

PARTIAL FRACTIONS

To express a single rational fraction as a sum of two or more single rational fractions which are called partial fractions. The process of expressing a rational fraction as a sum of partial fractions is called partial fraction resolution.

Exp : $\frac{3x}{(x-1(x+2)} = \frac{1}{x-1} + \frac{2}{x+2}$

The right hand side of eq is called partial fractions.

RATIONAL FRACTION

The quotient of two polynomials $\frac{\mathrm{P(x)}}{\mathrm{Q(x)}}$ where Q (x) ≠ O with no common fractions is called a rational fraction. A rational fraction is of two types.

1) Proper Rational Fraction

A rational fraction $\frac{\mathrm{P(x)}}{\mathrm{Q(x)}}$ is called proper rational fraction if the degree of the polynomial (P(x) in the numerator is less than the degree of the polynomial Q (x) in the denominator Exp: $\frac{3}{x+1}, \frac{2x+5}{x^2+4}, \frac{9x^2}{x^3-1}$

ii) Improper Rational Fraction

A rational fraction $\frac{\mathrm{P(x)}}{\mathrm{Q(x)}}$ is called an improper rational fraction if the degree of the polynomial P(x) in the numerator is equal or of greater than the degree of the polynomial Q(x) in the denominator. Exp $\frac{x}{2x-3}, \frac{x^2-3}{3x+1}$.

Any improper rational fraction can be reduced by division to a mixed form, consisting of the sum of a polynomial and a proper rational fraction.

Exp: $\frac{3x^2+1}{x-2} = 3x+6+\frac{13}{x-2}$

Resolution of a rational fraction $\frac{\mathrm{P(x)}}{\mathrm{Q(x)}}$ into partial fraction

i) The degree of P(x) must be less than that of Q(x). if not, divide and work with the remainder theorem.
ii) Clear the given equation of fractions.
iii) Equate the co-efficient of like terms (powers of x).
iv) Solve the resulting equations for the co-efficient.

Cases of partial fractions resolution

Case I: Resolution of $\frac{\text{P(x)}}{\text{Q(x)}}$ into partial fractions when Q(x) has only non-repeated linear factors. As these fractions have linear denominators their numerators contains only one constant. To find out these constants, first check degree of numerator, if it is improper rational fraction, then first change it in to proper rational fractions by division and resolve proper rational fraction in to partial fractions. Multiply the equation throughout by its LCM. Put denominator of each linear fraction equal to 0 and find value of x. then put that value of x into clear equation of fraction. Repeat that process for each unknown constants. Find values of all unknown constants and put it into partial fractions, which is the required form of equation.

Exp: $$\frac{7x+25}{(x+3)(x+4} = \frac{A}{x+3}+\frac{B}{x+4}$$
$$= \frac{4}{x+2}+\frac{3}{x+4}$$

ii) If denominator of rational fraction $\frac{\text{P(x)}}{\text{Q(x)}}$ is non linear but factorisable. Then first factorize it, reduce it into its simplest form, then change it in to partial fraction.

Exp: $$\frac{x^2-10x+13}{(x-1)(x^2-5x+6)} = \frac{x^2-10x+13}{(x-1)(x-2)(x-3)} = \frac{A}{x-1}+\frac{B}{x-2}+\frac{C}{x-3}$$
$$= \frac{2}{x-1}+\frac{3}{x-2}-\frac{4}{x-3}$$

Exp: $$\frac{2x^3+x^2-x-3}{x(2x+3)(x-1)} = 1+\frac{2x-3}{x(2x+3)(x-1)}$$

It is an improper fraction, first change it into mixed form by division. Then apply partial fraction resolution on proper rational fraction i.e.

$$\frac{2x-3}{x(2x+3)(x-1)} = \frac{A}{x}+\frac{B}{2x+3}+\frac{C}{x-1}$$

Hence partial fractions are: $1+\frac{1}{x}-\frac{8}{5(2x+3)}-\frac{1}{5(x-\ )}$

Case II

Resolution of $\frac{\text{P(x)}}{\text{Q(x)}}$ into partial fractions when Q(x) has repeated linear factors: If the denominator of the rational fraction has repeated linear factors, then each ascending power of linear factor in denominator has one constant in the numerators, and values of unknown. Constants in numerator can be found by comparing coefficients of same powers of x.

Exp: $$\frac{x^2+x-1}{(x+2)^2} = \frac{A}{x+2}+\frac{B}{(x+2)^2}$$
$$\frac{x^2+x-1}{(x2+)^2} = \frac{1}{x+2}-\frac{3}{(x+2)^2}$$

Case III:

Resolution of $\frac{P(x)}{Q(x)}$ into partial fractions when Q(x) contains non repeated irreducible quadratic factors:

A quadratic factor is irreducible if it cannot be written as the product of two linear factors with real coefficients. Exp: x^2+3, x^2+x+1. In this case the irreducible quadratic factor in denominator has two unknown constants in numerator, one constant with x and other independent of x. the values of unknown constants in numerators can be found by comparing coefficients of same power of x.

Exp: $\frac{3x-11}{(x^2+1)(x+3)} = \frac{A}{x+3} + \frac{Bx+1}{x^2+1}$

Multiply by lcm

$3x-11 = A(x^2+1) + (Bx+C)(X+3) \rightarrow$ (1)

$= (A+B)x^2+(3B+C)x + (A+3C)$

Put $x+3=0$, $x = -3$ in (1) then $A = -2$

Comparing Co-efficient of x^2 : $A+B = 0$, $B=2$

Comparing co-efficient of x : $3B+C = 3$, $C = -3$

$$\frac{3x-11}{(x^2+1)(x+3)} = \frac{-2}{x+3} + \frac{2x-3}{x^2+1}$$

Deduction

1. Partial fraction resolution is only applicable on proper rational fraction.
2. When a rational fraction is separated into partial fractions, the result is an identity. i.e. the co-efficient of like powers of variable are equal.
3. If denominator of any partial fraction is linear non – repeated function, the value of its unknown constant in numerator can be found by substituting value of x in the identity which can be got by putting each linear factor of the denominators equal to zero.