A-Level Mathematics Paper 1

Notes

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Title	A-L MATHEMATICS PAPER 1 NOTES
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Published by	MS Books (042-35774780)
Legal Advisor	Ashir Najeeb Khan (Advocate) **AKBAR LAW CHAMBERS** 39-40, 1st Floor, Sadiq Plaza, The Mall, Lahore. 0307-4299886, 042-36314839
For Complaints/Order	**MS Books** 83-B Ghalib Market, Gulberg III Lahore (042-35774780),(03334504507),(03334548651)

Contents

COORDINATE GEOMETRY

Syllabus

Candidates should be able to:	Notes and examples
• find the equation of a straight line given sufficient information	e.g. given two points, or one point and the gradient.
• interpret and use any of the forms $y = mx + c$, $y - y_1 = m(x - x_1)$, $ax + by + c = 0$ in solving problems	Including calculations of distances, gradients, midpoints, points of intersection and use of the relationship between the gradients of parallel and perpendicular lines.
• understand that the equation $(x - a)^2 + (y - b)^2 = r^2$ represents the circle with centre (a, b) and radius r	Including use of the expanded form $x^2 + y^2 + 2gx + 2fy + c = 0$.
• use algebraic methods to solve problems involving lines and circles	Including use of elementary geometrical properties of circles, e.g. tangent perpendicular to radius, angle in a semicircle, symmetry. Implicit differentiation is not included.
• understand the relationship between a graph and its associated algebraic equation, and use the relationship between points of intersection of graphs and solutions of equations.	e.g. to determine the set of values of k for which the line $y = x + k$ intersects, touches or does not meet a quadratic curve.

COORDINATE GEOMETRY

The study of points, straight lines and curves defined by algebraic expression is called coordinate geometry.

Distance between two points or length of line segment

If A(x_1,y_1) and B (x_2,y_2) are two points, then distance between A and B is AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint of a line segment

The coordinates of mid point of A (x_1, y_1) and B (x_2, y_2) are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

The diagonals of rectangle, rhombus, square and parallelogram have common mid points. To find out coordinate of fourth point of these shapes, when other three points are given, first supposed coordinate of unknown point (x,y), then put mid points of both diagram equal and find (x,y).

Gradient of straight line

Gradient is the measure of slope of line with respect to the positive x – axis.

The tangent of the angle counted anti clock wise, which a line make with positive x-axis is called gradient of line.

OR

The increase in the y – coordinate divided by the increase in the x- coordinate of two pints on the line is called gradient usually it is denoted by 'm' m = $\frac{y_2 - y_1}{x_2 - x_1}$

Properties of gradient

1. A line parallel to x-axis has gradient 0.
2. A line parallel to y-axis has gradient undefined. (∞)
3. If points are collinear (lie on same straight line) then their gradients are equal.
4. If two lines are parallel, then their gradients are equal. To find gradient of parallel line, first make 'y' as a subject and then take co-efficient of x as a gradient.
5. If two lines are perpendicular or normal, then product of their gradient is equal to -1, i.e. $m_1 \times m_2 = -1$
6. If gradient is +ive, then angle made by line with +ive x -axis is acute. If gradient is –ive, then angle of line with x-axis is obtuse.

Equation of straight line

1. Equation of straight line when its gradient (m) and y – intercept (c) is given y = mx+c. where x and y are the variables of the equation.
2. Equation of straight line when its gradient (m) and a point (x_1,y_1) is given OR equation of line passing through two points OR equation of tangent is $y-y_1 = m(x-x_1)$.
   To show that given straight line is tangent to the curve. First substitute equation of line in equation of curve, simplify it, then show that $b^2-4ac=0$.

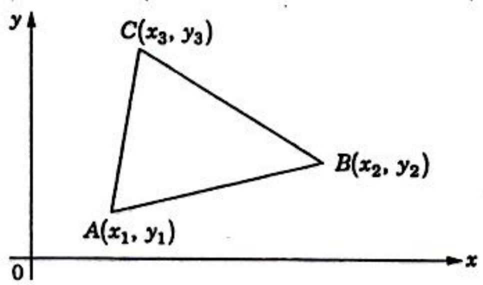
3. Equation of perpendicular or normal or perpendicular bisector is $y-y_1 = -\frac{1}{m}(x - x_1)$.

In case of perpendicular bisector (x_1, y_1) = mid point.

Use of graph to find unknown coordinates

- ❖ To find unknown point in case of two intersecting lines, find their point of intersection of two lines by elimination or by substitution method.
- ❖ To find point of intersection of a line and curve substitute value of any variable of linear equation in equation of curve and find x, y co-ordinate.
- ❖ In quadrilateral if one diagonal is line of symmetry of given figure, then point of intersection of diagonals will be the mid point of other diagonal.

AREA OF POLYGONS



Given vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, the area of ΔABC is

Area of ΔABC	$\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$ $= \frac{1}{2}\left\|(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\right\|$

} arrange the 3 points in anti-clockwise direction ends with the start point

arrange $\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$ in cross pattern |\\\|-|///|

Example 1

The ends of a line segment are $(p-q, p+q)$ and $(p+q, p-q)$. Find the length of the line segment, its gradient and the coordinates of its mid-point.

For the length and gradient you have to calculate

$$x_2 - x_1 = (p+q)-(p-q) = p+q-p+q = 2q$$

and $$y_2 - y_1 = (p-q)-(p+q) = p-q-p-q = -2q.$$

The length is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(2q)^2+(-2q)^2} = \sqrt{4q^2+4q^2} = \sqrt{8q^2}$.

The gradient is $\frac{y_2-y_1}{x_2-x_1} = \frac{-2q}{2q} = -1$.

For the mid-point you have to calculate

$$x_1 + x_2 = (p-q)+(p+q) = p-q+p+q = 2p$$

and $$y_1 + y_2 = (p+q)+(p-q) = p+q+p-q = 2p.$$

The mid-point is $\left(\frac{1}{2}(x_1+x_2), \frac{1}{2}(y_1+y_2)\right) = \left(\frac{1}{2}(2p), \frac{1}{2}(2p)\right) = (p, p)$.

Example

(using mid-points) In this method, begin by finding the mid-points
Prove that the points $A(1,1)$, $B(5,3)$, $C(3,0)$ and $D(-1,-2)$ form a parallelogram.
of the diagonals AC and BD. If these points are the same, then the diagonals bisect each other, so the quadrilateral is a parallelogram.

The mid-point of AC is $\left(\frac{1}{2}(1+3),\frac{1}{2}(1+0)\right)$, which is $\left(2,\frac{1}{2}\right)$. The mid-point of BD is $\left(\frac{1}{2}(5+(-1)),\frac{1}{2}(3+(-2))\right)$, which is also $\left(2,\frac{1}{2}\right)$. So $ABCD$ is a parallelogram.

Example

Find the equation of the line joining the points $(3,4)$ and $(-1,2)$.

To find this equation, first find the gradient of the line joining $(3,4)$ to $(-1,2)$. Then you can use the equation $y-y_1=m(x-x_1)$.

The gradient of the line joining $(3,4)$ to $(-1,2)$ is $\dfrac{2-4}{(-1)-3}=\dfrac{-2}{-4}=\frac{1}{2}$.

The equation of the line through $(3,4)$ with gradient $\frac{1}{2}$ is $y-4=\frac{1}{2}(x-3)$. After multiplying out and simplifying you get $2y-8=x-3$, or $2y=x+5$.