O-Level Add Maths

Notes

Edition 2020

Rafique Akthar Baloch

(0300-4897003)

Visiting Teacher

LACAS

Title O-L ADD MATHS NOTES

Author RAFIQUE AKTHAR BALOCH (0300-4897003)

Published by MS Books (042-35774780)

Legal Advisor Ashir Najeeb Khan (Advocate High Court)
AKBAR LAW CHAMBERS
39-40, 1st Floor, Sadiq Plaza, The Mall, Lahore
042-36314839, 0307-4299886

For Complaints/Order **MS Books**
83-B Ghalib Market, Gulberg III Lahore
(042-35774780),(03334504507),(03334548651)

Preface

This book has been designed for O Level Additional Mathematics students. It covers latest syllabus 2020 prescribed by CIE . I tried to make an attempt to present the material in a simple, clear and straightforward way.Three procedures are indispensable in the enjoyment and success in Mathematics: understanding, memorization and practice.The purpose of this book is to help the students in doing these. A large number of practice questions from past papers examination has been provided to help the students in learning of step-by-step procedure of problem solving. This will enable students to revise and practice systematically. Students are urged to grapple with these questions for acquiring solid understanding.

Thanks are due to the LACAS family for providing professional environment and colleagues to assist in logical thinking. I acknowledge with thanks the generous cooperation by Mr Aamir Mustafa, Mr Asghar Hayat,Mr Mahwar Mustafa, Mr Qamar Fayyaz,Mr Zahid Amin and my son Junaid Rafique. I regret any error and misprints and pledge to correct these in my next edition. I wish to express infinite gratitude to **MS Books** who took great pains for the publication of this work.

Sincerely Yours

Rafique Akhtar Baloch

(0300-4897003)

Salient Features of the Book

1. Each topic consists of basic skills and comprehensive notes which are helpful to solve the questions.
2. All topics of syllabus are arranged in a manner which keep the learner's interest alive.
3. There is penalty of practice questions at the end of each topic.
4. Answers of all practice questions are given at the end, so that the students can easily analysis their performance.
5. Useful tips are given to solve questions in minimum time.
6. Practice questions are given with marks allocation
7. At the end formulae sheet is given for complete preparation

Contents

`

1	Simultaneous Equations	5
2	Indices & Surds	10
3	Logarithms	23
4	Quadratic Expressions and Equations	35
5	Remainder & Factor Theorems	49
6	Coordinate Geometry	62
7	Linear Law	81
8	Functions	95
9	Binomial Theorem	122
10	Permutations & Combinations	132
11	Trigonometry	146
12	Circular Measure	170
13	Differentiation	186
14	Application of Differentiation	201
15	Integration	218
16	Application of Integration (Area under a curve)	234
17	Kinematics	247
18	Vectors	262
19	Series (A P & G P)	275

Simultaneous Equations

Simultaneous Linear Equations

- Simultaneous equations are equations that have common unknowns and therefore needed to be solved together.
- 2 simultaneous linear equations are of from:

2 simultaneous Linear equations	$a_1y + b_1x = c_1$ $a_2y + b_2x = c_2$	where x and y are common unknowns and a_1, a_2, b_1, b_2, c_1, c_2, are constants

- To solve 2 simultaneous linear equations, the method commonly used is elimination method (although substitution method can also be used):

(1) Elimination Method	**Step 1:** Eliminate one unknown first by equating (multiply or divide and cancelling out (add subtract) an identical term **Step 2:** Solve the equation for the remaining unknown **Step 3:** Substitute the solution obtained in (2) into any one of the equations to solve for the eliminated unknown (usually 1 solution to each to each unknown)

[Q] Solve the following simultaneous equations

$$3x - 4y = 11$$

$$7x - 5y = 4$$

Solution:

$3x - 4y = 11 \ldots\ldots\ldots (1)$
$7x - 5y = 4 \ldots\ldots\ldots (2)$
(1) X 5: $15x - 20y = 55 \ldots\ldots\ldots (3)$
(2) X 4: $28x - 20y = 16 \ldots\ldots\ldots (4)$
(4) – (3): $(28x - 20y) - (15x - 20y) = 16 - 55$

$\Rightarrow 28x - 15x = -39$

$\Rightarrow 13x = -39$

$\therefore x = -3$

Substitute x = -3 into (1) $3(-3)-4y = 11$

$\Rightarrow -9 - 4y = 11$

$\Rightarrow -4y = 20$

$\therefore y = -5$

$\therefore x = 3, Y = -5$ **Ans.**

NON LINEAR EQUATIONS

- Any equation not in the form of $\boxed{ay + bx = c}$ is considered non linear equation.
- When we deal with non-linear equations, there can be more than one solution. Elimination method may not able to help us obtain the answer.
- A pair of simultaneous equations, one linear and one non-linear, or both non-linear,are usually solved using substitution method:

2 Substitution method	**STEP 1** Express one of the unknowns in terms of the other unknown in one of the equations to form a new expression (use linear equation first, if there is one) **STEP2** Substitute this new expression into unused equation and solve for the unknown **STEP3** Substitute the solution(s) obtained in 2 into the new expressions to solve for the other unknown

When we solve 2 simultaneous equations, the solution are actually the intersection points (x and y coordinates) of the 2 intersecting curves (or straight lines) rearea……..? The simultaneous equations.

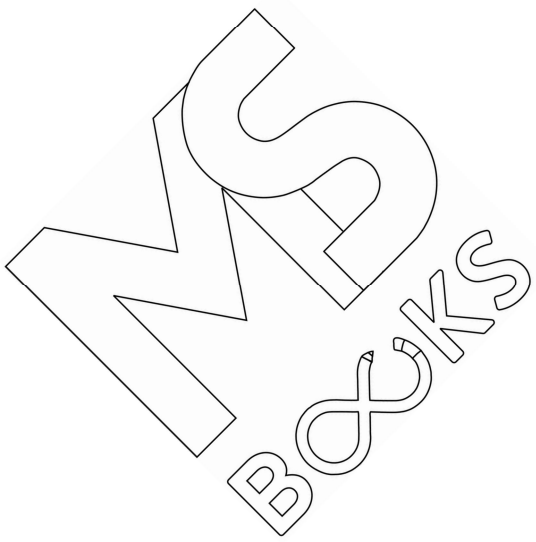
[Q] Solve the following simultaneous equations:

$$3x + 4y = 2$$
$$x^2 + 8xy + 12 = 0$$

SOLUTION

$3x + 4y = 2$…….. (1)
$x^2 + 8xy + 12$ …….. (2)

From (1): $3x + 4y = 2$
$\Rightarrow 3x = 2 - 4y$
$\Rightarrow x = \frac{2-4y}{3}$…… (3)



Substitute (3) into (2):

$$x^2 + 8xy + 12 = 0$$

$$\Rightarrow \left(\frac{2-4y}{3}\right)^2 + 8\left(\frac{2-4y}{3}\right)y + 12 = 0$$

$$\Rightarrow \frac{4-16y+16y^2}{9} + \frac{16y-32y^2}{3} + 12 = 0$$

X9: $\Rightarrow 4 - 16y + 16y^2 + 3(16y - 32y^2) + (9)(12) = 0$

$\Rightarrow 4 - 16y + 16y^2 + 48y - 96y^2 + 108 = 0$

$\Rightarrow -80y^2 + 32y + 112 = 0$

$\Rightarrow 5y^2 - 2y - 7 = 0$

$\Rightarrow (5y - 7)(y + 1) = 0$

$\Rightarrow 5y - 7 = 0$ or $y + 1 = 0$

$\therefore y = \frac{7}{5}$ $\therefore y = -1$

Substitute $y = \frac{7}{5}$ into (3): $x = \frac{2-4\left(\frac{7}{5}\right)}{3}$

$$= -\frac{18}{5} \times \frac{1}{3}$$

$$= -\frac{6}{5}$$

$$= -1\frac{1}{5}$$

Substitute $y = -1$ into (3): $x = \frac{2-4(-1)}{3}$

$$= -\frac{6}{3}$$

$$= 2$$

$\therefore x = -1\frac{1}{5}$, $y = 1\frac{2}{5}$ or $x = 2$, $y = -1$ **(Ans)**

Past Paper Questions

1 Find the coordinates of the points where the straight line $y = 2x - 3$ intersects the curve $x^2 + y^2 + xy + x = 30$. [5]

2 The straight line $5y + 2x = 1$ meets the curve $xy + 24 = 0$ at the points A and B. Find the length of AB, correct to one decimal place. [6]

3 Solve the simultaneous equations $5x + 3y = 2$ and $\frac{2}{x} - \frac{3}{y} = 1$. [5]

4 Solve the simultaneous equations

$$x + 3y = 13,$$

$$x^2 + 3y^2 = 43.$$ [5]

5 Find the coordinates of the points of intersection of the curve $y^2 + y = 10x - 8x^2$ and the straight line $y + 4x + 1 = 0$. [5]

6 The line $3x + 4y = 15$ cuts the curve $2xy = 9$ at the points A and B. Find the length of the line AB. [6]

7 The line $y = x - 5$ meets the curve $x^2 + y^2 + 2x - 35 = 0$ at the points A and B. Find the exact length of AB. [6]

8 Solve the simultaneous equations

$$2x^2 + 3y^2 = 7xy,$$

$$x + y = 4.$$ [5]

9 **(i)** Calculate the coordinates of the points where the line $y = x + 2$ cuts the curve $x^2 + y^2 = 10$. [4]

10 The line $x - 2y = 6$ intersects the curve $x^2 + xy + 10y + 4y^2 = 156$ at the points A and B. Find the length of AB. [7]

11 Find the coordinates of the points where the line $2y = x - 1$ meets the curve $x^2 + y^2 = 29$. [5]

12 The straight line $2x + y = 14$ intersects the curve $2x^2 - y^2 = 2xy - 6$ at the points A and B. Show that the length of AB is $24\sqrt{5}$ units. [7]

13 The line $x + y = 10$ meets the curve $y^2 = 2x + 4$ at the points A and B. Find the coordinates of the mid-point of AB. [5]

14 Find the distance between the points of intersection of the curve $y = 3 + \frac{4}{x}$ and the line $y = 4x + 9$. [6]

15 Find the coordinates of the points where the line $2y - 3x = 6$ intersects the curve $\frac{x^2}{4} + \frac{y^2}{9} = 5$. [5]

Answers

1 $(3, 3)$ and $(-1, -5)$

2 $(-\frac{15}{2}, \frac{16}{5})$ and $(8, -3)$ 16.7 units

3 $\left(\frac{1}{5}, \frac{1}{3}\right)$ and $(4, -6)$

4 $(2.5, 3.5)$ and $(4, 3)$

5 $(0, -1)$ & $\left(\frac{1}{4}, -2\right)$

6 $A\left(2, \frac{9}{4}\right)$ and $B\left(3, \frac{3}{2}\right)$ = 1.25 units

7 $(-1, -6)$ and $(5, 0)$ $= 6\sqrt{2}$

8 $(3, 1)$ and $(\frac{4}{3}, \frac{8}{3})$

9 (i) $(-3, -1)$ and $(1, 3)$ (ii) $-\sqrt{\frac{3}{2}}$ or $\sqrt{\frac{3}{2}}$

10 A is $(-6, -6)$ and B is $(10, 2)$ $= 17.9$

11 $(-\frac{23}{5}, -\frac{14}{5})$ and $(5, 2)$

12 A is $(5, 4)$ and $B = (-19, 52)$.

13 $A(16, -6)$, $B(6, 4)$ $(11, -1)$

14 $\left(\frac{1}{2}, 11\right)$ and $(-2, 1)$ 10.3 units

15 $(2, 6)$ and $(-4, -3)$